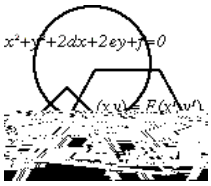


1. Let S be the set of all integers of the form $P^2 - 1$ where P is a prime number greater than 5. Let N be the largest integer that divides every member of S. Find, with proof, the value of N.

2. In triangle ABC, $(\cos B)(\cos C) = \cos A$. Find, with proof, the numerical value of $(\tan B)(\tan C)$.

3. Suppose that $n^2 - 1$ boys are lined up shoulder-to-shoulder from left to right in a straight line. Prove that it is always possible to select n + 1 boys to take one step forward so that, going from left to right, their heights are either consistently increasing or consistently decreasing.

4. The lengths of the sides of triangle ABC are in the ratio of 4:5:6. The bisector of the largest angle of the triangle is drawn. Prove that one of the two triangles formed also has sides whose lengths are in the ratio of 4:5:6.

5. All the factors of the polynomial $P(x) = x^3 + ax^2 + bx + b$ are linear with integer coefficients, and neither a nor b are zero or one. Find all possible pairs (a, b), and prove that you have found them all.

3 3 ± 3 6LQFH 3 LV D SULPH DQG 3 ! 3 LV RG
DQG 3 PXVW EH FRQVHFXWLYH HYHQ QXPEHUV 7K
PXOWLSOH RI 3 ZKLFK LV D SULPH E\ 3$ DQG 3 VLQFH 3 ±
WKUHH FRQVHFXWLYH LQWHJHUV RQH RI WKHP PXVW
HLWKHU 33 ± PXVW EH D PXOWLSOH RI 3 VLQFH 7EKHUHIRUH
,I 33 ,I 3 3 6LQFH LV WKH JFG RI DQ
N=24 ODUJHVW

RV% FRV& %FRVRV&± V%Q VLQ&±

± FRV% FRV& VLQ% VLQ&

7KHUHIRUH FRV% FRV& VLQ% VLQ& VR WKDW

WKDW

$VVXPH LW LV LPSRVVLEOH WR ILQG Q ER\V LQ DVR
:H ZLOO VKRZ WKDW LW LV WKHQ SRVVLEOH WR ILQG Q
6WDUWLQJ ZLWK WKH ILUVW ER\ ZH IRUP D ³FOXE´ LQ V
ER\ WDOOHU WKDQ KLP WKH ILUVW ER\ WDOOHU WKDQ
WKDQ Q ER\V LQ WKLV FOXE 1H[W ZH FUHDWH D VHFR
WKH ILUVW FOXE DQG FKRRVLQJ WKH ILUVW ER\ ZKR LV
ILUVW ER\ LQ WKH VHFRQG FOXE DQG FRQWLQXLQJ LQ W
ER\V LQ WKH VHFRQG FOXE 1RWH WKDW HDFK ER\ LQ V
ER\ LQ WKH ILUVW FOXE ZKR LV WDOOHU WKDQ KLP RU
1RZ IURP DPRQJ WKH ER\V ZKR DUH QRW LQ HLWKHU R
FOXE VWDUWLQJ ZLWK WKH ILUVW ER\ QRW LQ WKH ILU
WKDQ Q ER\V LQ WKH WKLUG FOXE DQG HDFK RI WKHVH
ZKR LV WDOOHU WKHQ KLP :H FRQWLQXH LQ WKLV ZD\
PRUH WKDQ Q PHPEHUV DQG HDFK PHPEHU RI HDFK FO
FOXE

7KHUH DUH PHPEHUV LQ WKHVH Q FOXEV VR QRZ OHW X
DQ\ RI WKHVH FOXEV +H PXVW IROORZ D WDOOHU ER\
WK

/HW WKH VLGHV RI WULDQJOH $%& KDYH OHQJWKV
ORVV RI 6LQH 7KHRUHP OHW WKH ORQJHVW VLGH RI
LV WKH ODUJHVW DQJOH %L'QWHUFHSWRU WKHRUHP ELVHFWRU
$& DW SRLQW ' 6LQFH WKH OHQJWKV RI FRUUHVSRQGLQ
WR LPLODUDWLHV DUH SURSRUWLRQDO ZH QHHG RQO\ SURYH
WKDW WKH QHZ OHQJWKV IRUP $%& LV VLPLODU WR
6LQFH ERWK WULDQJOHV $%& DQG $'% DUH DFXWH ZH
WKDW WWUKLDQJOHZR DUH VLPLODU

0HWKRG

8VLQJ WKH DQJOH–ELVHFWRU WKHRUHP ZKLFK
–[ 6LQFH WKH UDWLR RI $% WR $' LV

$% $& 7KXV $' % DQG $& KDYH WZR SDLUV
$' $%
RI SURSRUWLRQDO VLGHV DQG VKDUH WKH LQFOXGHG
DQJOH $ 7KHUHIRUH WKH WZR WULDQJOHV DUH VLP
SUR%Y%QDOV KDV VLGHV WZKRVH OHQJWKV
LQ WKH UDWLR RI

0HWKRG

6LQFH $% DQG $& VKDUH DQJOH $ ZH QHHG RQO\ ILQG
RQH DGGLWLRQDO SDLU RI FRQJUXHQW DQJOHV $Q
FDQQRW EH FRQJUXHQW WR DQJOH $%& ,I LW LV FR
WR DQJOH & WKHQ WULDQJOH %'& ZRXOG KDYH WR
7KLV FDQ KDSSHQ LI WKH PHDVXUH RI DQJOH $%&
ZLOO EH WKDW RI DQJOH & $

8VLQJ WKH /DZ RI &RVLQHV RQ

FRV% –

FRV FRV– –

FRV FRV – 7KHUHIRUH P &P

7KXV WULDQJOH $%& LV VLPLODU WR WULDQJOH $'% DQG
OHQJWKV WKH UDWLR RI