



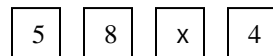
THE 2011-2012 KENNEBEC VALLEY REGIONAL HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

Each of the following 25 questions is worth 4 points. The total score is out of 100 points. Do not use a calculator. Each question has 5 possible answers. Only one answer is correct. You may use a pencil.

NO CALCULATORS

90 MINUTES



- In the puzzle at the right, the number in each empty square is obtained by adding the two numbers in the row directly above. For example, $5 + 8 = 13$. What is the value of x ?
 (A) 2 (B) 3 (C) 6 (D) 7 (E) 9
- A circle passes through the points $(0, 0)$, $(0, 2)$ and $(4, 0)$. What is the area of this circle?
 (A) 5 (B) 8 (C) 9 (D) 10 (E) 16
- Tom found the value of $3^{21} = 10,4A0,353,20B$. He found all the digits correctly except the fourth and last digits, denoted by A and B, respectively. What is the value of A?
 (A) 0 (B) 2 (C) 3 (D) 6 (E) 8
- In determining standings in a certain hockey league, a team receives 3 points for each win, 1 point for each tie, and -1 point for each loss. After playing 50 games, the Ducks have a total of 76 points. How many more wins than losses do the Ducks have at this time in the season?
 (A) 12 (B) 13 (C) 14 (D) 15 (E) 16
- Let $x = m + n$ where m and n are positive integers satisfying $2^m + m^n = 2^7$. The
 (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

7. If the measure of $\angle ABE$ is 6 degrees greater than the measure of $\angle DCE$, compute the number of degrees in the measure of $\angle FD$.
- (A) 6 (B) 8 (C) 10 (D) 12 (E) 16
8. If q and r are the zeros of the quadratic polynomial $x^2 + 15x + 31$, find the quadratic polynomial whose zeros are $q + 1$ and $r + 1$.
- (A) $x^2 + 17x + 31$ (B) $x^2 + 15x + 33$ (C) $x^2 + 13x + 17$
(D) $x^2 + 19x + 37$ (E) None of these
9. The makers of Delight Ice Cream put a coupon for a free ice cream bar in every 80th bar they make. They put a coupon for 2 free bars in every 180th bar and a coupon for 3 free bars in every 300th bar. If they put all three coupons in every n^{th} bar, compute n .
- (A) 1200 (B) 1800 (C) 2400 (D) 3600 (E) 5400
10. Starting at opposite ends of a straight moving walkway at an airport, which travels at a constant rate of k ft/sec, Don and Debbie walk towards each other (Don moving in the direction the walkway is moving, Debbie moving against the direction the walkway is moving). They meet at a point one-seventh of the way from one end of the walkway. If they were on a normal (non-moving) floor they would each walk at a rate of 3 feet per second. Determine the value of k .
- (A)

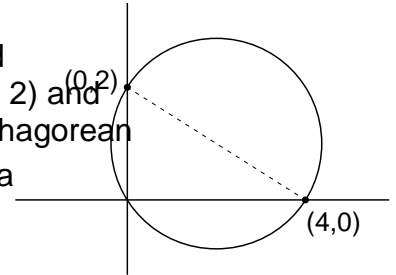
20. The number 2011 can be written as $a^2 - b^2$ where a and b are integers. Compute the value of $a^2 + b^2$.
- (A) 2018041 (B) 2022061 (C) 2024072 (D) 2026085 (E) 2033051
21. Consider the following system of equations: (1) $ax + by = c$ and (2) $dx + ey = f$ ($c \neq 0, f \neq 0$). When $x = 0$, equation (1) yields $y = 3$ and (2) yields $y = 6$. When $y = 0$, (1) yields $x = -3$ and (2) yields $x = 3$. What is the common solution (x, y) for the system?
- (A) (1, 2) (B) (2, 6) (C) (4, 1) (D) (6, 2) (E) (1, 4)
22. Rectangle ABCD has sides of length 3 and 4. Rectangle PCQD is similar to rectangle ABCD, with P inside rectangle ABCD. Compute the distance from P to AB.
- (A) 1 (B) $\frac{4}{3}$ (C) $\frac{7}{5}$ (D) $\frac{21}{17}$ (E) $\frac{27}{25}$
23. Let $S(n) = [\sqrt{1}] + [\sqrt{2}] + \dots + [\sqrt{n}]$ where $[k]$ is the greatest integer less than or equal to k . Compute the largest value of $k < 2011$ such that $S(2011) - S(k)$ is



SOLUTIONS

1. **A** The entries in the four empty boxes, from top to bottom and left to right $x + 8$, $x + 4$, $x + 21$ and $x + 12$. Then $(x + 21) + (x + 12) = 39$ or $2x + 33 = 39$, and $x = 2$.

2. **A** Because the angle at $(0, 0)$ is a right angle, it is inscribed in a semicircle, which makes the segment connecting $(0, 2)$ and $(4, 0)$ a diameter. Using the distance formula (or the Pythagorean Theorem), the diameter of the circle is $\sqrt{20}$, making the area $\frac{1}{4}(\sqrt{20})^2 = 5$.



3. **D** Of course, one could compute the value of 3^{21} directly, but that would take some time and might lead to careless errors. A more general approach is as follows. Let's find B first. The powers of 3, taken in order from 3^1 end in the repeating pattern 3, 9, 7, 1. Since 21 is one more than a multiple of 4, $B = 3$. Since 3^{21} is a multiple of 9, its digits must sum to a multiple of 9. Since the known digits and B have a sum of 21, the missing digit A must be 6.

4. **B** Let W = the number of wins, T = the number of ties, and L = the number of losses. $W + T + L = 50$ and $3W + T - 2L = 101$. Solving these equations, we find $W = 24$, $T = 1$, and $L = 25$.

7. **A** Represent $m'FBC$ as $180 - (x + 6) = 174 - x$. Also,
 $m'DCE = m'BCF = x$. Then $m'AFD = 180 - (174 - x) - x = 6$.

8. **C** We could find the zeros of the given polynomial, increase each by 1, and use them to find the answer. However, that is time consuming. Here are two shorter methods.

Method 1: IC BT /TT0 1 C BT /TT0 1 T4</MCID 6 >>BDC 0 -1.15 TD (8)Tj (.)L0k0 scn Tf

13. A $\log_2 4 = 2$ and $\log_2 2 = \frac{1}{2}$. Using the triangle inequality, we have

$$\log_3 x < 2 + \frac{1}{2} \quad \text{and} \quad \log_3 x + \frac{1}{2} > 2$$

Therefore, $\log_3 x < \frac{5}{2} \quad \checkmark \quad x < 3^{\frac{5}{2}}$ or $x < 9\sqrt{3}$ and

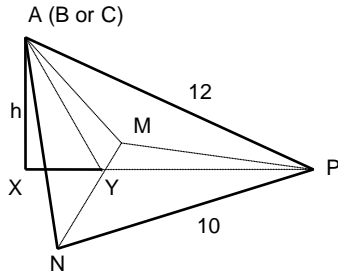
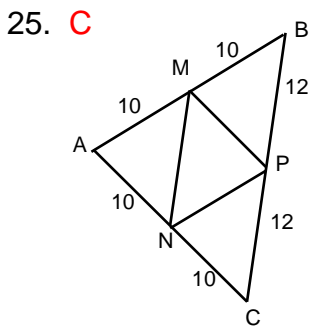
$$\log_3 x + \frac{1}{2} > 2 \quad \checkmark \quad \log_3 x > \frac{3}{2} \quad \text{or} \quad x > 3\sqrt{3}.$$

Therefore, the set of all possible values of x is $3\sqrt{3} < x < 9\sqrt{3}$. Since $3\sqrt{3} \approx 5.2$ and $9\sqrt{3} \approx 15.6$, choice A (5) is the only choice that is not possible.

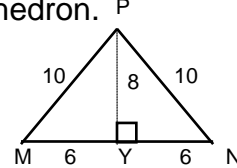
18. **E** Using $x = 5$, we obtain $2f(5) + f(-4) = 25$
Using $x = -4$, we obtain $2f(-4) + f(5) = 16$.
Multiplying the first equation by 2 and subtracting the equations we obtain
 $-3f(5) = -34$ from which $f(5) = \frac{34}{3}$.
19. **E** Let m be the slope of the line tangent to the ellipse. The equation of the tangent

23. **D** Since $\sqrt{1936} = 44$ and $\sqrt{2025} = 45$, all numbers from $\lceil\sqrt{1936}\rceil$ to $\lfloor\sqrt{2011}\rfloor$ must equal 44. If $K \leq 1936$, $S(2011) - S(K) = \lfloor\sqrt{2011}\rfloor + \lfloor\sqrt{2010}\rfloor + \dots + \lfloor\sqrt{K+1}\rfloor = 44(2011 - K) = (4)(11)(2011 - K)$. Therefore $S(2011) - S(K)$ will be a perfect square for $2011 - K = 11$, and $K = 2000$.

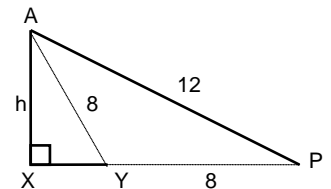
24. **C** & D
 Before the operation, the sum of the squares of these two is $2a^2 + 2a + 1 = 2a^2 + 2$, whereas after the operation the sum of the squares is simply $2a^2$. Since the other numbers do not change, we see that the sum of the squares of all nine numbers goes down by two during every operation. Because $(6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2 + 0^2 + 0^2) = 182$, Anne performed $182 \div 2 = 91$ operations



We need to find the area B of triangle MNP (whose sides are 10, 10, and 12) and the length of the altitude of the tetrahedron.



It is easy to see that the area of triangle MNP is $\frac{1}{2}(8)(12)$ or $B = 48$ square units



Next we find the length of h . In the middle diagram above, triangle PYA has sides $PA = 12$, $PY = 8$ and $AY = 8$.

Thus, triangle PYA is obtuse, as shown. Using the Law of Cosines on triangle PYA.

$$144 = 64 + 64 - 128 \cos(\angle APY) \quad \text{and} \quad \cos(\angle AYP) = \frac{1}{8}$$

Therefore, $\cos(\angle AYP) = \frac{1}{8}$, making $XY = 1$ and $h = 3\sqrt{7}$. Hence, $V = \frac{1}{3}Bh = \frac{1}{3}(48)(3\sqrt{7}) = 48\sqrt{7}$

The desired ordered pair is $(48, 7)$