THE 2017–2018 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

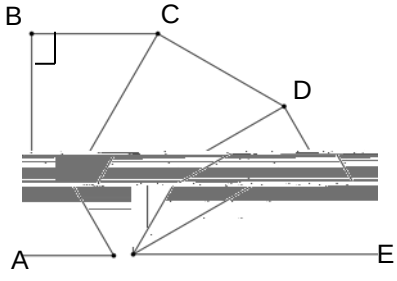
NO CALCULATORS

90 MINUTES

1. Huckleberry High School held a dance for its students. The first N students paid $\$10$ each for a ticket to the dance. But then the ticket price was reduced. After the price reduction, the number of students buying tickets was 50% more than before the price reduction, but the amount of money received by the school was only 20% more than before the price reduction. What was the amount of the reduced ticket price?

   (A) $\$8.00$ (B) $\$7.50$ (C) $\$7.20$ (D) $\$7.00$ (E) $\$6.40$

2. The absolute value of a certain number is N



6. The integers b and c are chosen so that

(i) one of the roots of the quadratic equation $5x^2 + bx + c = 0$ is 2 and

(ii) one of the roots of the quadratic equation $5x^2 + cx + b = 0$ is 3.

Compute the sum of the other two roots.

(A) $\frac{7}{5}$ (B) $\frac{8}{5}$ (C) $\frac{1}{5}$ (D) $\frac{7}{5}$ (E) $\frac{8}{5}$

7. If $\log(K) + (\log 4)(\log 4) = (\log 40)(\log 40)$, compute K.

(A) 40 (B) 64 (C) 80 (D) 128 (E) 160

8.

13. For how many positive integers n will $\frac{(n-1)^2}{n-12}$ be an integer.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. In the diagram, two circles of radii 1 and 2 are tangent to each other and the positive x-axis at the origin. A segment that is tangent to the smaller circle is drawn from point B, the y-intercept of the larger circle, to point A on the positive x-axis. If the coordinates of point A are $(a,0)$, compute the value of a.

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19. Consider the following infinite series

$$S = 1 - 2 - 3 + 4 + 5 - 6 - 7 + 8 + 9 - 10 - 11 + 12 + 13 - 14 - 15 + \ldots$$

Define the partial sums of the series as follows:

$S_1 = 1$, $S_2 = 1 - 2$, $S_3 = 1 - 2 - 3$, $S_4 = 1 - 2 - 3 + 4$, $S_5 = 1 - 2 - 3 + 4 + 5$,

and so on. For what value of n does S_n exceed 2017 for the first time?

(A) 2017 (B) 2018 (C)) 2019 (D) 2020 (E) 2021

20. Each of the following numbers is a prime, and all but one of them can be expressed as the difference of the

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Solutions

1. A Let x = the reduced ticket price. Then 1.2(10) = 1.5x. Solving, $x = \frac{12}{1.5} = 8$.

2. C Let N = the number. N is negative, for if it wasn't, its absolute value would not be more than N. Therefore, –N = N + 3.5 and N = –1.75 = $\frac{7}{4}$. Thus, taking the reciprocal increases N by $\frac{4}{7}$, $\frac{7}{4}$, $\frac{33}{28}$.

3. C Only days 11 and 22 can be the middle two digits (since no month has 33 days). Since there are 12 months in a year, the first two digits can only be 01, 02, 03, …, 12. Thus there are 24 possibilities.

4. D The number of groups of four students is $\frac{!s}{4!(\quad 4)!}$

8. C Since consecutive angles of a parallelogram are supplementary, each angle of the parallelogram has the same sine.
Let K = the area of the parallelogram. Then

$K = 7x = 4(20 - x)$ 03125C C

12. C

17. E

22. C Let n be the number of people who voted and a be the number who voted for D by the time when he had 45% of the vote. Then

$$\frac{45}{100} = \frac{9}{20}$$

Using the Pythagorean Theorem on BEC and solving for y, we obtain $y = \frac{\sqrt{4x^2 - 75}}{2}$.

Since AC must be an integer, and $DE = \frac{5}{2}$, $4x^2 - 75$ must be the square of an integer.

Trying each of the choices,

$4(13^2) - 75 = 601$ is not a perfect square.

$4(15^2) - 75 = 825$ is not a perfect square.

$4(17^2) - 75 = 1081$ is not a perfect square.

$4(19^2) - 75 = 1369 = 37^2$.

Therefore, the only possible choice is D.