THE 2019 – 2020 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For [(1CHOr [(1Cu[(1Cu[(Q   E(E)-2)11(t(E)-roM)-roMa5yC6 [(Ma5y Ma5yow)15(iMa5yng 26 [(5 ques)7t(E)-riN

6. Suppose that a and b are positive integers, x is a real number, and i is the imaginary unit.
   If $:= E$

13.   If $t \{ y_{\bar{o}}$ is a factor of $y \{ t_{\bar{o}}$ (where $t \{ y_{\bar{o}}$

20. The first term of an arithmetic sequence of distinct terms is 4. The 1st, 5th, 15th and kth terms of the arithmetic sequence form a geometric sequence in the same order. Compute the value of k.

   (A) 25　　　(B) 35　　　(C) 36　　　(D) 39　　　(E) 40

21. For which of the following values of n will $\dfrac{n^2+5\,4\,7+9}{n}$ be an integer, while $\dfrac{n^2+5\,4\,7+9}{n}$ is not?

   (A) 2079　　(B) 3575　　(C) 5136　　(D) 6237　　(E) None of these

22. Let p, q, and r be the roots of the equation $T^7 - s\,T^6 + t\,T - v = r$. Compute the value of the expression $\left(1-\dfrac{1}{p}\right)\left(1-\dfrac{1}{q}\right)\left(1-\dfrac{1}{r}\right)$.

   (A) $\dfrac{7}{2}$　　(B) $\dfrac{1}{4}$　　(C) -3　　(D) -12　　(E) $\dfrac{29}{2}$

23. In triangle ABC, AB = 7, BC = 33, and AC = 37.
   A circle centered at A with radius AB intersects
   ray CB at point D and side AC at point E, as shown.
   Compute the

Solutions

1. **A** Each of the three rows has four 1x2 rectangles. Each of the five columns has two 1x2 rectangles. There are a total of four 2x4 rectangles. The total is 26.

2 . **E** Since 2 is the only even prime number, the smallest number in set A is $2 + 2019 = 2021$.
All the rest of the numbers in A are the sum of two odd numbers and are, therefore, even.
The smallest number in set B is $(2)(2019) = 4038$, and all the rest of the numbers in B are
odd. Thus $A \cap B$ is empty and the number of elements in the intersection is 0.

3. **D** $1 + 2 + 3 + D = 360$ and $4 + 5 + E = 180$.
Adding these two equations:
$1 + 2 + 3 + 4 + 5 + D + E = 540$. Also $D + E = 180 \pm x$.
Therefore, $1 + 2 + 3 + 4 + 5 + 180 \pm x = 540$ from which
$1 + 2 + 3 + 4 + 5 = 360 \mp x$.

4. **D** $6^{2 \cdot 3y}$ $\dfrac{6^2}{6^{3y}}$ $\dfrac{36}{6^{3y}}$ 2. From this, 6^{3y} 18. Therefore, $x^{6>7i}$ L : x^6 ; : x^{7i} ; L : u x; : s z, L x v z

5. **B** Let $T = $ # of families with twins, $R = $ # of families with triplets, and $Q = $ # of families with
quadruplets. Then we are given $T + R + Q = 26$ and $2T = 3R = 4Q$. From the second equation
$T = \dfrac{3}{2}R$ and $Q = \dfrac{3}{4}R$. Therefore, $\dfrac{3}{2}R + R + \dfrac{3}{4}R = 26$ and $R = 8$.

6. **B** When := E > E^7 is expanded, the real part is ^{7} F u=> ^{6}. Therefore,
= ^{7} F u=> ^{6} L =: = ^{6} F u> ^{6} ; L F y vä Since a and b are positive integers, and is a factor
of 74, a little trial and error gives $a = 1$ and $b = 5$ as the only solution, and a $\pm$ b

10. **B** Let the correct two‐digit score be $10A + B$. Then the mis‐entered score was $10B + A$. Since the class average was 2.7 points less than it should have been, and there are 20

V W X G H Q W V   L Q   W K-H   F O D V V $ E E \ ¶ V   P L V V

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24. **B**   « $\frac{(48)(49)}{2}$ = (24)(49). Let n = the number of integers from A to B,

inclusive. Thus $n < 48$ and the sum of the integers from A to B, inclusive, is $\frac{n}{2}(A + B)$.

Therefore, $6[\frac{n}{2}(A + B)] = (24)(49)$ Ÿ $3n(A + B) = :t^7::u::y^6;$ Ÿ $n(A + B) = :t^7::y^6;$

Since $n < 48$, the only possible values of n are 2, 4, 7, 8, 14, and 28.

Letting n equal each of these values leads to the following results

  $n = 2$ A