

**THE 2022 2023 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION**

PART II

Calculators are NOT permitted

Time allowed: 2 hours

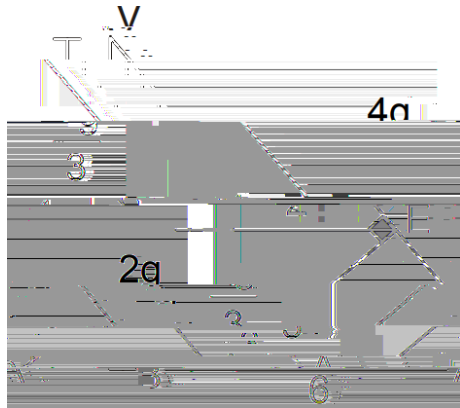
1. If n is an even positive integer and $2022^{2023} + 2023^{2022}$ is a perfect square, prove that n is a multiple of 8.

2. Prove $2022^{2023} + 2023^{2022} \equiv n^2 \pmod{2022}$ for any integer n .

3. If $f(x) = x^2 - 2x + 1$ – Find the minimum value for:

4. In the diagram, we have a right circular cylinder, where; AB is perpendicular to CD , EF is perpendicular to GH and the intersection point is E, and M is the center of the upper base.

Solutions



Notice on triangle :
 triangle is similar to , then

$$\frac{\quad}{\quad} = \frac{\quad}{\quad}$$

So,

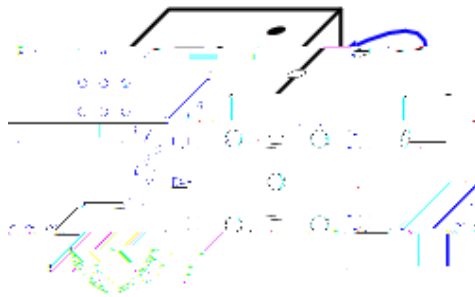
$$\quad = \quad$$

5. (A, B, C, D, E, F, G, H), where G is the cube completely hidden at the bottom left back:

We can observe:

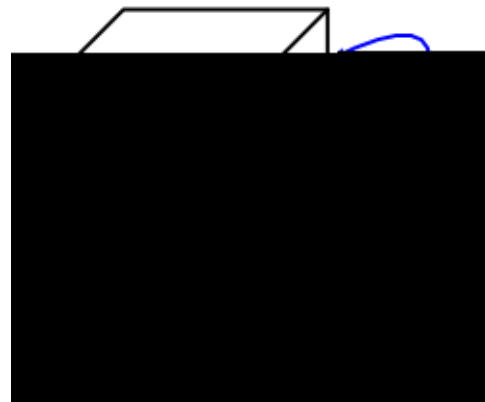
- From cube A we can conclude that on the left face of cube B must be 7 dots.
- Cube F has faces with 3 and 6 dots adjacent to each other, then B must have 6 dots in the faces opposite to the one with 5 dots.
- So then (because there are 1, 3, 4, 5, 6 or 7 dots in each face) the bottom face in cube B has 4 dots.

Summary for **cube B**:



Also, from previous analysis we can conclude:

- Cube A has 6 dots on the left face.
- Cube F has 1 dot on the bottom face.
- Cube D has 6 dots on the hidden face.
- Cube H** has two possibilities:



So, the minimum number of dots on the hidden face for cube H is 4.

