

THE 2023–2024 KENNESAW STATE UNIVERSITY

marked. Note that wild guessing is likely to lower your score. When the exam is over, hand in your answers to your proctor. You may keep your copy of the questions.

NO CALCULATORS

1. A number is *Beprisque* if it is the only integer between a prime number and a perfect square.



13. One of the roots of the polynomial equation  $x^3 + px + q = 0$  is  $\sqrt[3]{2} + \sqrt[3]{5}$ . If  $a$  and  $b$  are rational numbers, compute the value of  $b - a$ .
- (A) 3      (B) 7      (C) 21      (D) 23      (E) 29
14. Compute the value of  $\log_{64} \frac{5}{6} \cdot \log_{67} \frac{5}{6} \cdot \log_{68} \frac{5}{6} \cdot \log_{66} \frac{5}{6} \cdot \log_{67} \frac{5}{6}$ .
- (A) -2023      (B) -1      (C) 1      (D) 2023      (E) None of these
15. If  $z^6 = (1 + i)^6$  where  $i$  is the imaginary unit and  $r > 0$  and  $\theta$  are positive integers, compute  $r$ .
- (A) 9      (B) 10      (C) 11      (D) 12      (E) 13
16. The first two positive integers for which  $1 + 2 + 3 + \dots + n$  is a perfect square are 1 and 8.

20. In the magic square shown, each row, column, and main diagonal sum to 100, where T, R, I, A, N, G, L, E

## Solutions

1. **D** Listing the perfect squares less than 100, it is easy to identify the Beprisque numbers as 2, 3, 8, 10, 24, 48, 80, 82, for a total of 8.
  
2. **C** There are 2 possibilities shown,

(N)  $0^6 = 2$  (D) 33. However, (C) ? and (E)  $-3$ . Thus, (B)  $6 = 2 + 33$   
and (A)  $-12$

10. C Let  $p$  = number of pennies,  $2p$  = number of dimes, and  $6p$  = number of quarters. Then the value of the coins in cents is  $p + 20p + 150p = 171p$ . The amount of money in the bag must be divisible by 171. Only choice (C) works.

11. B Let  $U = B^2 \{ T \}$ . Then,  $\frac{i > 5}{i ? 5}$

17. B Let  $x$  equal the area of  $\triangle ABO$ . Since the area of  $\triangle BCA$  is 2, it follows that the area of  $\triangle AEC$  is  $3x$ . Similarly, since  $\triangle ABD$  has area 1, we see that  $\triangle ADO$  has area  $3x$ . Finally, since  $\triangle DAC$  has area 3, we conclude that the area of  $\triangle AEC$  is  $3(3x) = 9x$ . In particular, the area of  $\triangle AEC$  is  $4x + (2 + x) = 4$ . Now  $\triangle ABO$  and  $\triangle BCO$  share the same altitude to  $AC$ , so their areas are proportional to the lengths of their bases, namely  $AO$  and  $OC$ . Similarly, the areas  $\triangle ADO$  and  $\triangle CDO$  are also proportional to  $AO$  and  $OC$ . Thus  $\frac{ES}{SG} = \frac{e}{6-e} = \frac{5-e}{6-e} \Rightarrow 2x + 1 = 5x + 2$ , from which  $x = \text{area of } \triangle ABO = \frac{6}{9}$ .

18. A Since  $a_1, a_2, a_3$  are in arithmetic sequence,  $F_7 = F_6 + F_5$  and  $F_8 = 2F_6 - F_5$ . Therefore,  $F_6, 2F_6 - F_5$  are in geometric sequence which gives us  $\frac{F_6}{F_5} = \frac{2F_6 - F_5}{F_6}$ . Now,  $2F_6 - F_5, \frac{F_6}{F_5}$  are in harmonic sequence, which gives us  $\frac{6}{2F_6 - F_5}, \frac{F_6}{F_5}, \frac{5}{F_6}$  are in arithmetic sequence. Therefore,  $\frac{5}{F_6} = \frac{5}{2F_6 - F_5}$ . Hence  $F_5 = 5 = (2F_6 - F_5) \Rightarrow F_6 = \frac{5}{3}$ . Thus,  $F_7 = \frac{5}{3}$ .

19. E =

21. **D** Let the coordinates of the other vertex of the longer leg be  $(x, y)$ . Then



Noting that  $\cos^6 15^\circ - \sin^6 15^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$  and  $\cos^6 15^\circ + \sin^6 15^\circ = 1$ , this last equation becomes  $3T^6 - 4\sqrt{3}T + 3 = 0$