THE 2024 2025 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I   MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

1. **Problem.** On every twig of a tree, there are six leaves. On every branch of the tree, there are three twigs and three additional leaves. On every bough of the tree, there are two branches, two additional twigs, and two additional leaves. If the tree has six boughs, how many leaves does it have?

   (A) 216   (B) 300   (C) 324   (D) 336   (E) 354

2. **Problem.** Aaron, Barbara, and Cece bought the same kinds of flour and sugar from the same grocery store. Aaron bought twice as much flour as Barbara, but Barbara bought twice as much sugar as Aaron, and Aaron paid 25% more than Barbara. If Cece bought as much flour

8. **Problem.** If you leave a Poisson ® brand light bulb on continuously for 10 days, the probability is 19% that it burns out. If you leave two Poisson ® brand light bulbs on continuously for 15 days, what is the probability that at least one of them burns out, rounded to the nearest percentage point?

Assume that the probability that a light bulb burns out in any time interval depends only on the length of the time interval, not on how long the light bulb has been left on before then.

(A) 42% (B) 47% (C) 52% (D) 57% (E) 67%

9. **Problem.** The equation $_{1/}$ 3 − $_{1/4}$ = 1 has two positive real solutions. What is their product?

(A) 3/4 (B) 1 (C) 4/3 (D) 3 (E) 4

10. **Problem.** ^{11} on the board, but Elle copied it down

digits. Which of the following is the true value of 11^{11}?

(A) 283,511,760,611
(B) 285,311,670,611
(C) 285,311,760,161
(D) 285,311,766,011
(E) 285,317,160,611

11. **Problem.** An operation is defined on the positive real numbers by the formula

$$= \frac{}{+},$$

where C>0, but the value of C is unknown. We say that is **commutative** if x y = y x for all positive real numbers x and y; we say that is **associative** if x (y z) = (x y) z for all positive real numbers x, y, and z. Which of the following is true?

(A) Without knowing the value of C, we cannot say whether is associative or commutative.
(B) is associative no matter what C is, but without knowing the value of C, we cannot say whether is commutative.
(C) is commutative no matter what C is, but without knowing the value of C, we cannot say whether is associative.
(D) is both associative and commutative no matter what C is.
(E) None of the above.

15. **Problem.** The diagram below shows connections between several computers in a network; every computer can send messages to any other computer, possibly relayed through some other computers.

For every connection in the network, a coin is flipped; if it lands heads, nothing is changed, but if it lands tails, the connection is removed. What is the probability that after this is done, every computer can still send messages to any other computer?

(A) 1/4096 (B) 1/16 (C) 3/32 (D) 81/256 (E) 91/128

16. **Problem.** ^{2024} + (1- ^{2024} simplifies to an integer. What is the last digit of that integer?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

17. **Problem.** A fair 6-sided die has 2 red faces and 4 blue faces. Ray and Bea play a game. Ray rolls the die repeatedly until one of two things happens:

1. The die lands on a red face twice, not necessarily consecutively, or
2.

18. **Problem.** The floor in a very large room is tiled with many hexagonal, triangular, and square tiles in a regular pattern; a fragment of the tiling pattern is shown below.

(A) $\frac{1}{3}$ (B) $\frac{\sqrt{3}-1}{2}$ (C) $1-\frac{\sqrt{3}}{3}$ (D) $2\sqrt{3}-3$ (E) $\frac{1}{2}$

19. **Problem.** Rina, Rohan, and Ryan go running in the same park at the same time, but notu

n nth

Solutions

1. **Solution.** The correct answer is (D) 336.

   Every branch has 3×6+3 = 21 leaves on it. Every bough has 2×21 + 2×6 + 2 = 56 leaves on it. Therefore, the tree has 6×56 = 336 leaves.

2. **Solution.** The correct answer is (C) 20% more.

   x for flour and y for sugar. Then Barbara paid only $x/2$ for flour, but $2y$ for sugar; we are given that $x+y=1.25(x/2+2y)$. Solving for x, we get $x=4y$.

   Cece paid x for flour and $2y$
   $(x+2y)/(x+y)$, or $(4y+2y)/(4y+y) = 1.2$; therefore, Cece paid 20% more than Aaron.

3. **Solution.** The correct answer is (A) $\frac{1}{4}$.

   Method 1: We could, of course, solve the problem by rewriting the fraction as $\frac{1332}{5328}$ and then simplifying. But there is an easier way. In both sums, each digit appears twice in the hundreds place, twice in the tens place, and twice in the ones place, so altogether it is multiplied by 200+20+2. Therefore the numerator is (1+2+3)(200+20+2), the denominator is (7+8+9)(200+20+2), and the factor of 200+20+2 cancels: the fraction can be simplified to $\frac{1+2+3}{7+8+9}$ or $\frac{1}{4}$.

   Method 2: The numbers can be paired up like 123+321=444, 132+312=444, ... ,89+987 = 1776,798+978=1776, ... So, the answer is (3 * 444) / (3 * 1776) = 1/4.

4. **Solution.** The correct answer is (A) Peter.

   The most direct answer is to observe that Reese *must* be telling the truth: otherwise,

5. **Solution.** The correct answer is (E) $c > a > b$.

To compare a and b, write a as 2^{1024}; this is more than $2^{1001} = (2^7)^{143} = 128^{143}$. On the other hand, $b = 125\ 124\ 123 \quad 3\ 2\ 1$; this is the product of 125 factors, all at most 125, so $b < 125^{125}$. Since $128^{143} > 125^{143} > 125^{125}$, $a > b$.

To compare a and c, write a as 2^{1024}; this is less than $2^{1050} = (2^3)^{350} = 8^{350}$. On the other hand, c can be written as 3^{729}; this is more than $3^{700} = (3^2)^{350} = 9^{350}$. Since $8^{350} < 9^{350}$, $a < c$.

6. **Solution.** The correct answer is (E) $276\ 20^4$.

Since $(x + 20)(x - 20) = x^2 - 20^2$, we can rewrite the polynomial as $(x^2 - 20^2)^{24}(x - 20)$. In the first factor, $(x^2 - 20^2)^{24}$, there are only even powers of x, so we need to take an x from the final factor of $(x - 20)$, and an x^{44} from $(x^2 - 20^2)^{24}$. This comes from the term with $(x^2)^{22}(-20^2)^2$, whose coefficient is $\frac{24\ 23}{2} = 276$ by the binomial theorem.

7. **Solution.** The correct answer is (D) $\frac{36}{5}$.

must be equal; in other words, 6(XY+4) = 7(XZ+4). Points Y and Z give us similar

9. **Solution.** The correct answer is (E) 4.

We can rewrite $_{1/}$ 3 as $-\frac{3}{}$ and $_{1/4}$ as $-\frac{}{4}$, whea68ea68ea68<</MCI1 0 0 1 350.59 677.5

12. **Solution.** The correct answer is (A) It is less than 10% of the speed of a car.

k times

faster than Carmen. Then a car going the same way as Carmen will pass her if it passes

- 1/*k*; any later than that, and Carmen will

reach school first. On the other hand, a car going the opposite way will pass Carmen if it

-1/*k* and 1; any earlier than that, and it will reach

Therefore Carmen must be seeing (1+1/*k*)/(1 - 1/*k*) = (*k*+1)/(*k*-1) more cars going the opposite direction. Setting (*k*+1)/(*k*-1) = 6/5 and solving, we get *k* = 11. Therefore a car is

of a car.

13. **Solution.** The correct answer is (E) 5000/10001.

The exact zigzagging pattern does not matter; as long as the ant visits all 10000 squares

15. **Solution.** The correct answer is (B) 1/16.

In each triangle of computers, there is a ½ probability that at least two connections are removed, and a ½ probability that at most one connection is removed.

If none of the triangles lost more than one connection (which happens with probability $(½)^4$, or 1/16), then each triangle can still communicate to the central computer, and therefore all the computers can still send messages to each other.

If, on the other hand, there is a triangle that lost two or more connections, then not all computers can communicate: either one of the outer computers in that triangle cannot send messages to anywhere, or else the two outer computers in that triangle can send messages to each other, but nowhere else. So the scenario in the previous paragraph, which happens with probability 1/16, is the only way that all computers can communicate with all others.

16. **Solution.** The correct answer is (D) in the case of 2024

<u>Method I</u>: One of the ways to solve the problem is to observe the repetition in the last digit

^{n} + (1- ^{n} ^{n} as $a + b$ $^{n+1}$ = $(a+6b) + (a+b$

and we can keep track of the units digit of a and b: starting from (1,1) when n=0, they go

$^n = a + b$ ^{n} +

(1- $^n = 2a$

22. **Solution.** The correct answer is (B) 5 : 4.

The value of a_n is simply 8^{n-1}: at each stage, the area is multiplied by 8, since we take 8 copies of the previous stage. The value of p_n is more complicated. There is an outer perimeter of 4 3^{n-1}: the fractal fits inside a 3^{n-1} by 3^{n-1} square at the n^{th}

24. **Solution.** The correct answer is (D) 7.

The square originally numbered 64 stays in place as we fold the paper, and we can number the other cells of the grid by the number of the folding step (between 1 and 6) which places them on top of that square:

2	6	6	4	4	6	6	1
5	6	6	5	5	6	6	5
5	6	6	5	5	6	6	5
3	6	6	4	4	6	6	3
3	6	6	4	4	6	6	3
5	6	6	5	5	6	6	5
5	6	6	5	5	6	6	5
2	6	6	4	4	6	6	

The numbers 1, 3, 5, 7, 9 end up placed on top of the bottom right square in steps 2, 6, 4, 6, 5, respectively. So 3 and 7 (the numbers that are placed on top of the bottom right square in step 6) are closer to the top of the grid.

To decide between 3 and 7, we watch what happens to them closely. After fold 2, they end up above square 63 in the grid, with 3 above 7. Then, they stay put until fold 6, which puts them both above square 64, and also reverses their order. Therefore 7 is above 3 in the stack.

25. **Solution.** The correct answer is (D) 49.

x,y). First, |x|+|y| can

x+y

(0,0) by going up, up, and down;
(0,2) by going up, down, and up;
(0,4) by going down, up, and up;
(0,6) by going up, up, and up;
(1,1) by going right, down, and up;
(1,3) by going left, right, and up;
(1,5) by going right, up, and up;
(2,2) by going down, right, and up;
(2,4) by going up, right, and up;
(3,3) by going right, right, and up.